Non-Gaussianity of one-point distribution functions in extended Lagrangian perturbation theory

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Abstract. We study the one-point probability distribution functions (PDFs) of the peculiar velocity and the density fluctuation in a cosmological fluid. Within the perturbative approach to the structure formation scenario, the effect of "pressure" has recently been an area of research interest. The velocity dispersion of the cosmological fluid creates effective "pressure" or viscosity terms. From this viewpoint, because the pressure reflects a nonlinear effect of the motion of the fluid, the pressure model would include nonlinear effects. Here we analyze the Lagrangian linear perturbation PDFs for both the Zel'dovich approximation and the pressure model. We find that the PDFs of the peculiar velocity remain Gaussian, even if we consider the pressure. For the PDFs of the density fluctuation, the occurrence of non-Gaussianity depends on the "equation of state" for the fluid. Therefore we distinguish the "equation of state" using the PDFs.

PACS numbers: 04.25.Nx, 95.30.Lz, 98.65.Dx

arXiv:astro-ph/0504209v2 20 Apr 2005

1. Introduction

The scenario for the formation of large-scale structure in the Universe is based on the gravitational instability of the primordial density fluctuation. This fluctuation may have been generated from the quantum fluctuations during the inflation phase [1, 2, 3]. The fluctuation is spontaneously grown by its self-gravitational instability. To describe the evolution of the density fluctuation, many approaches have been carried out.

For a perturbative approach, the Lagrangian description provides a relatively accurate model even in a quasi-linear stage. Zel'dovich [4] originally proposed a linear Lagrangian approximation for dust (pressureless) fluid. This approximation is called the Zel'dovich approximation (ZA) [1, 2, 4, 5, 6, 7, 8, 9, 10, 11]. ZA describes the evolution of density fluctuation better than the Eulerian approximation [12, 13, 14]. Although ZA gives an accurate description until a quasi-linear regime develops, ZA cannot describe the model after the formation of caustics. In ZA, even after the formation of caustics, the fluid elements keep moving in the direction set up by the initial condition. In order to proceed with a hydrodynamical description without the formation of caustics, although modified models, such as the "adhesion approximation" [15] and the "Truncated Zel'dovich approximation" [16, 17] were proposed, the physical origin of the modification has not yet been clarified, the physical origin of the modification has not yet been clarified.

Recently, instead of the dust fluid, pressure-supported fluid has been considered. The collisionless Boltzmann equation [18] describes the motion of matter in phase space. The basic equations of hydrodynamics are obtained by integrating the collisionless Boltzmann equation over velocity space. In past approximations, such as ZA and its modified models, velocity dispersion was ignored. Buchert and Domínguez [19] argued that the effect of the velocity dispersion should be noticeable beyond the caustics. They showed that when the velocity dispersion can be considered isotropic, it gives effective "pressure" or viscosity terms. Furthermore, they argued for a relation between mass density ρ and pressure P, i.e., an "equation of state." If the relation between the density of matter and pressure can be regarded as barotropic, they showed that the "equation of state" should take the form $P \propto \rho^{5/3}$. Buchert *et al.* [20] showed how the viscosity term is generated by the effective pressure of a fluid under the assumption that the peculiar acceleration is parallel to the peculiar velocity. Domínguez [21, 22] introduced the idea that the evolution equations for the matter fields are smoothed over a smoothing length; then the viscosity term in the "adhesion approximation" can be derived by the expansion of coarse-grained equations. Recently, Buchert and Domínguez [23] discussed the origin of the viscosity term and the extension of the Lagrangian perturbation theory.

Departing from these points of view, the extension of the Lagrangian perturbation theory to cosmological fluids with pressure has been considered. Adler and Buchert [24] formulated the Lagrangian perturbation theory for a barotropic fluid. Morita and Tatekawa [25] and Tatekawa *et al.* [26] solved the Lagrangian perturbation equations for a polytropic fluid up to the second order. Recently, Tatekawa [27] solved these same equations up to the third order. Hereafter, we call this model the "pressure model." Buchert and Domínguez [23] call this model the "Euler-Jeans-Newton" (EJN) model.

In this paper, we study the occurrence of non-Gaussianity in the one-point probability density functions (PDFs). For one-point PDFs of the smoothed density and velocity fields in a cosmological model, Kofman *et al.* [28] analyzed the evolution beginning with a Gaussian initial fluctuation. Their analytic results are based on ZA. They found that the PDF of the peculiar velocity, both Eulerian and Lagrangian, remains Gaussian under linear approximation. For the density fluctuation, they showed that the PDF develops a shape similar to a lognormal distribution.

We consider PDFs as follows. From the point at which the pressure is derived from the collisionless Boltzmann equation (for example, [19, 20, 21, 22]), even if we consider only linear perturbation, the pressure model would include nonlinear effects. Therefore, even if we analyze the evolution of the peculiar velocity PDF only with linear approximation, it will deviate from Gaussian. Please note that the contribution of the pressure can be distinguished with PDFs. The effect of the pressure changes the shape and the evolution of the PDFs.

We analyze non-Gaussianity of the PDF of the peculiar velocity and density fluctuation. For our analyses, we compute skewness and kurtosis, which are statistical quantities for non-Gaussianity of the PDF. First, we show the results for the dust model with ZA and N-body simulation. Our results coincide with those of previous efforts. Then we indicate that the skewness and the kurtosis are useful for the analysis of non-Gaussianity. Next we analyze the PDFs for the pressure model. Here we consider both the Eulerian and the Lagrangian linear approximations. In the pressure model, because the growing rate of the fluctuation depends on the scale, the occurrence of the non-Gaussianity was expected. However, under linear approximation, the effect of the pressure does not contribute to occurrence of the non-Gaussianity very much. The PDF of the peculiar velocity remains largely Gaussian during evolution. For the PDF of the density fluctuation, the Eulerian linear approximation does not indicate the occurrence of non-Gaussianity. On the other hand, in the Lagrangian linear approximation non-Gaussianity does obviously occur. Furthermore, the evolution of the PDF depends on the "equation of state." Therefore if we analyze carefully the PDFs obtained from observations, we can expect to find a constraint to the "equation of state" for the cosmological fluid.

This paper is organized as follows. In Sec. 2, we briefly present the evolution equation for cosmological fluid. For simplicity, we consider only the Einstein-de Sitter Universe model.

In Sec. 3, we analyze non-Gaussianity of the one-point PDFs. Here we introduce two statistical quantities, skewness and kurtosis. First, in Sec. 3.1, we analyze the dust model. According to past study, the PDF of the density fluctuation develops a shape similar to a lognormal distribution. Its non-Gaussianity could be detected with the skewness and kurtosis. In Sec. 3.2, we analyze the pressure model. In the dust model, the PDFs in the Eulerian linear approximation remains Gaussian. Also in the pressure model, this tendency was unchanged. The PDF of the density fluctuation in the Lagrangian linear approximation was sensitive to the "equation of state."

In Sec. 4, we discuss our results and state conclusions.

2. The evolution of fluctuation for the cosmological fluid

Here we briefly introduce the evolution equation for cosmological fluid. In the comoving coordinates, the basic equations for cosmological fluid are described as

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot \{\boldsymbol{v}(1+\delta)\} = 0\,, \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla_x)\boldsymbol{v} + \frac{\dot{a}}{a}\boldsymbol{v} = \frac{1}{a}\tilde{\boldsymbol{g}} - \frac{1}{a\rho}\nabla_x P\,, \tag{2}$$

$$\nabla_x \times \tilde{\boldsymbol{g}} = \boldsymbol{0}\,, \tag{3}$$

$$\nabla_x \cdot \tilde{\boldsymbol{g}} = -4\pi G\rho_b a\delta\,, \tag{4}$$

$$\delta \equiv \frac{\rho - \rho_b}{\rho_b}\,. \tag{5}$$

In the Eulerian perturbation theory, the density fluctuation δ is regarded as a perturbative quantity. In linear approximation, from equation (1), we obtain

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot \boldsymbol{v} = 0\,. \tag{6}$$

Then we take a divergence of equation (2) and substitute equation (6) to it. Finally we obtain the evolution equation for the density fluctuation in the Eulerian linear approximation [29]:

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} - 4\pi G\rho_b\delta - \frac{1}{\rho_b a}\nabla_x^2 P = 0\,. \tag{7}$$

When we assume a polytropic fluid ($P = \kappa\rho^\gamma$), equation (7) becomes

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} - 4\pi G\rho_b\delta - \frac{\kappa\gamma\rho_b^{\gamma-1}}{a^2}\nabla_x^2 \delta = 0\,. \tag{8}$$

On the other hand, in the Lagrangian perturbation theory, the displacement from homogeneous distribution is considered.

$$\boldsymbol{x} = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q}, t)\,, \tag{9}$$

where $\boldsymbol{x}$ and $\boldsymbol{q}$ are the comoving Eulerian coordinates and the Lagrangian coordinates, respectively. $\boldsymbol{s}$ is the displacement vector that is regarded as a perturbative quantity. From equation (9), we can solve the continuous equation (1) exactly. Then the density fluctuation is given in the formally exact form.

$$\delta = 1 - J^{-1}, \quad J \equiv \det\left(\frac{\partial x_i}{\partial q_j}\right)\,. \tag{10}$$

J means the Jacobian of the coordinate transformation from Eulerian $\boldsymbol{x}$ to Lagrangian $\boldsymbol{q}$. Therefore when we derive the solutions for $\boldsymbol{s}$, we can know the evolution of the density fluctuation.

The peculiar velocity is given by

$$\boldsymbol{v} = a\dot{\boldsymbol{s}}\,. \tag{11}$$

To solve the perturbative equations, we decompose the Lagrangian perturbation to the longitudinal and the transverse modes:

$$\boldsymbol{s} = \nabla S + \boldsymbol{s}^T\,, \tag{12}$$

$$\nabla \cdot \boldsymbol{s}^T = 0\,, \tag{13}$$

where ∇ means the Lagrangian spacial derivative. Hereafter we consider only the longitudinal mode.

Using the Lagrangian displacement, we obtain the first-order perturbative equation.

$$\nabla^2\left(\ddot{S}^{(1)} + 2\frac{\dot{a}}{a}\dot{S}^{(1)} - 4\pi G\rho_b S^{(1)} - \frac{\kappa\gamma\rho_b^{\gamma-1}}{a^2}\nabla^2 S^{(1)}\right) = 0\,. \tag{14}$$

Let us compare equations (8) and (14). In both of the equations, the same operator acts on the perturbative quantity [25]. Therefore in the linear approximation, the form of the Lagrangian solutions is the same as that of the Eulerian solutions.

The first-order solutions for the longitudinal mode depend on spacial scale. Therefore the solutions are described with a Lagrangian wavenumber $\boldsymbol{K}$. For simplicity, in this paper, we discuss only perturbative solutions in the Einstein-de Sitter Universe model [25, 26].

$$\widehat{S}^{(1)}(\boldsymbol{K},t) = C^+(\boldsymbol{K})D^+(\boldsymbol{K},t) + C^-(\boldsymbol{K})D^-(\boldsymbol{K},t)\,, \tag{15}$$

$$D^{\pm}(\boldsymbol{K},t) = \begin{cases} t^{-1/6}\mathcal{J}_{\pm 5/(8-6\gamma)}(A|\boldsymbol{K}|t^{-\gamma+4/3}) & \text{for} \quad \gamma \neq \frac{4}{3}\,, \\ t^{-1/6\pm\sqrt{25/36-B|\boldsymbol{K}|^2}} & \text{for} \quad \gamma = \frac{4}{3}\,, \end{cases} \tag{16}$$

$$A \equiv \frac{3\sqrt{\kappa\gamma}(6\pi G)^{(1-\gamma)/2}}{|4-3\gamma|}\,, \; B \equiv \frac{4}{3}\kappa(6\pi G)^{-1/3}\,,$$

where $\mathcal{J}$ means Bessel function. $C^{\pm}(\boldsymbol{K})$ is given by the initial condition.

In this model, the behavior of the solutions strongly depends on the relation between the scale of fluctuation and the Jeans scale. Here we define the comoving Jeans wavenumber as

$$K_{\rm J} \equiv \left(\frac{4\pi G a^2}{\kappa\gamma\rho_{\rm b}^{\gamma-2}}\right)^{1/2}\,. \tag{17}$$

It depends on the ratio between the comoving Jeans wavenumbers and the wavenumber of the fluctuation whether the fluctuation grows. When we ignore the pressure term, we obtain the solutions of ZA.

$$S^{(1)} = t^{2/3}S_+(\boldsymbol{q}) + t^{-1}S_-(\boldsymbol{q})\,. \tag{18}$$

$S_{\pm}(\boldsymbol{q})$ is given by the initial condition.

3. Non-Gaussianity of the one-point distribution functions

We analyze the non-Gaussianity of the PDF of the peculiar velocity and the density fluctuation. To investigate non-Gaussianity, we compute two statistical quantities [3, 28, 30].

$$\text{skewness}: \left\langle \left(\frac{p-\langle p\rangle}{\sigma_p}\right)^3 \right\rangle ,$$
$$\text{kurtosis}: \left\langle \left(\frac{p-\langle p\rangle}{\sigma_p}\right)^4 \right\rangle - 3 ,$$
$$\sigma_p \equiv \left\langle (p-\langle p\rangle)^2 \right\rangle , p\text{: physical quantity} .$$

The skewness and the kurtosis shows display asymmetry and a non-Gaussian degree of "peakiness." If the distribution is completely Gaussian, both the skewness and the kurtosis become zero.

For simplicity, we set the Gaussian density field with a scale-free spectrum:

$$\mathcal{P}(k) \propto k . \tag{19}$$

To avoid divergence of the density fluctuation, we introduce a top-hat cutoff at $k^{-1} = 1h^{-1}\mathrm{Mpc}$ (at $a = 1$) in comoving Eulerian coordinates.

The Gaussian initial condition is generated by COSMICS [33]. We set up the initial condition at $a = 10^{-3}$. The initial peculiar velocity and the density fluctuation are adjusted by the growing solution of ZA. Both the skewness and the kurtosis of the initial condition are less than 10^{-2}.

For N-body simulation, we execute P^3M code. The parameters of simulation were given as follows:

$$\begin{aligned} &\text{Number of particles} : N = 128^3 , \\ &\text{Box size} \qquad\qquad\quad : L = 128h^{-1}\mathrm{Mpc} \quad (\text{at } a = 1) , \\ &\text{Softening Length} \quad\; : \varepsilon = 0.05h^{-1}\mathrm{Mpc} \quad (\text{at } a = 1) . \end{aligned}$$

For perturbative models, we set the parameters as follows:

$$\begin{aligned} &\text{Number of grids} : N = 128^3 , \\ &\text{Box size} \qquad\quad\;\; : L = 128h^{-1}\mathrm{Mpc} \quad (\text{at } a = 1) . \end{aligned}$$

The dispersion, skewness and kurtosis is computed by numerical method. For analyses of non-Gaussianity, we coarsen the fields. For the density fluctuation, we coarsen the density field over $1h^{-1}\mathrm{Mpc}$ (at $a = 1$) in comoving Eulerian coordinates with a top-hat window function. For the peculiar velocity, we analyze on grids at intervals of $1h^{-1}\mathrm{Mpc}$ (at $a = 1$) in Lagrangian space for Lagrangian perturbation models. In N-body simulation, we analyze the peculiar velocity of each particle.

We compute the dispersion, skewness and kurtosis at ten time slices from $a = 0.1$ to $a = 1.0$. The time interval is given as $\Delta a = 0.1$.

3.1. Non-Gaussianity in the dust model

First, we analyze the dust model. Here we analyze the PDF of the peculiar velocity and the density fluctuation for the Eulerian linear approximation, ZA and N-body simulation. For the dust models, Kofman *et al.* [28] analyzed one point PDFs in detail. They proved that for the PDF of the peculiar velocity, the Lagrangian PDF is equal to the Eulerain PDF at all times. For the PDF of the density fluctuation, Padmanabhan and Subramanian [31] derived the distribution function in the nonlinear regime using ZA.

Now we analyze the quasi-nonlinear stage. Figure 1 shows the dispersion of the density fluctuation. In our model, the dispersion becomes $\sigma > 0.1$ at $a = 0.1$. Because of shell-crossing, the growth of the density fluctuation becomes slow gradually.

Figure 2 shows the PDF of the density fluctuation in N-body simulation. During evolution, the PDF approaches log-normal distribution. The result coincides with that of the past analyses [28, 31, 32]. Figure 3 shows the PDF of the peculiar velocity. Because we set up almost isotropic initial condition, even if we pay attention to the one direction, we think that the generality of the result is not lost. Therefore, we choose the x-direction peculiar velocity. During evolution, because the dense region attracts surrounding matter, the probability of fast velocity increases. At $a = 1.0$, both the density fluctuation and the peculiar velocity obviously show non-Gaussian distribution. Therefore the skewness and the kurtosis must have non-zero value.

Figure 4 shows the skewness and the kurtosis of the density fluctuation. In N-body simulation, the PDF of the density fluctuation is well-fitted by log-normal distribution [28]. Therefore N-body simulation realizes strongly nonlinear evolution, then the skewness and the kurtosis increases rapidly. In ZA, the skewness and the kurtosis also increases during evolution. However, ZA can describe only quasi-nonlinear evolution, and after shell-crossing, i.e., the formation of the caustic, ZA cannot describe the evolution of nonlinear structure. Therefore the evolution of the skewness and the kurtosis stops gently.

Figure 5 shows the skewness and the kurtosis of the peculiar velocity. Because we set initial condition by almost isotropic distribution, the initial PDF of one direction of the peculiar velocity is Gaussian. As a result the skewness and the kurtosis increase rapidly. In ZA, the skewness and the kurtosis also increase during evolution. However, ZA can describe only quasi-nonlinear evolution, and after shell-crossing, i.e., the formation of the caustic, ZA cannot describe the evolution of nonlinear structure. Therefore the evolution of the skewness and the kurtosis stops gently. In the Eulerian linear approximation, the PDF of the peculiar velocity and the density fluctuation remains Gaussian at all times.

From our analyses of the PDF in the dust model, we can conclude that both the skewness and the kurtosis are useful quantities for analyses of non-Gaussianity. Next we adopt these analyses for the pressure model.

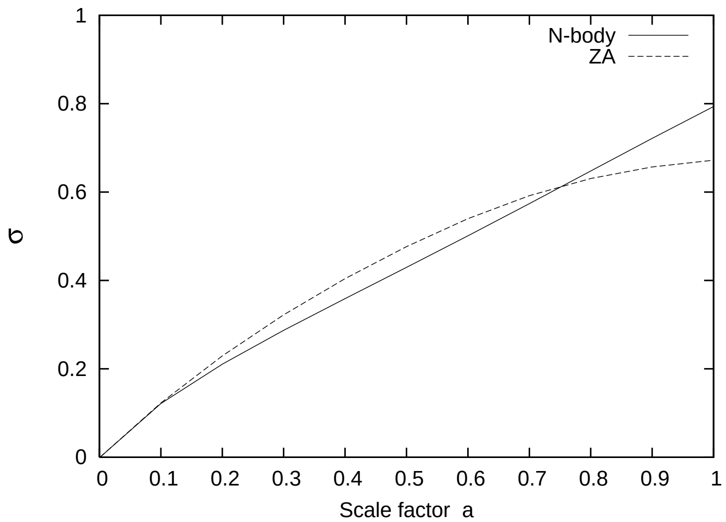

Figure 1. The dispersion of the density fluctuation in ZA and N-body simulation. Because of shell-crossing, the growth of the density fluctuation becomes slow gradually.

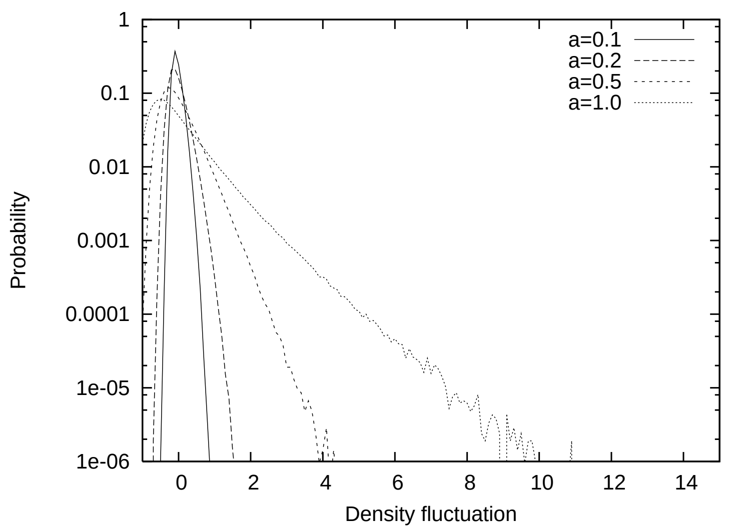

Figure 2. The PDF of the density fluctuation in N-body simulation. During evolution, the PDF approaches lognormal distribution.

3.2. Non-Gaussianity in the pressure model

In this subsection, we analyze the pressure model. Here we choose the polytropic index $\gamma = 4/3, 5/3$. In a previous paper [34], we compared the density field between N-body simulation and the Lagrangian approximations. In this comparison, we showed that if a small value was chose for the initial Jeans wavelength K_J (equation (17)), it is hard for a nonlinear structure to form the pressure suppress the evolution of the density fluctuation. On the other hand, if we choose a large K_J value, the evolution of the density fluctuation becomes almost same as that in the dust model. Therefore if we want to consider the effect of the pressure and the formation of the nonlinear structure,

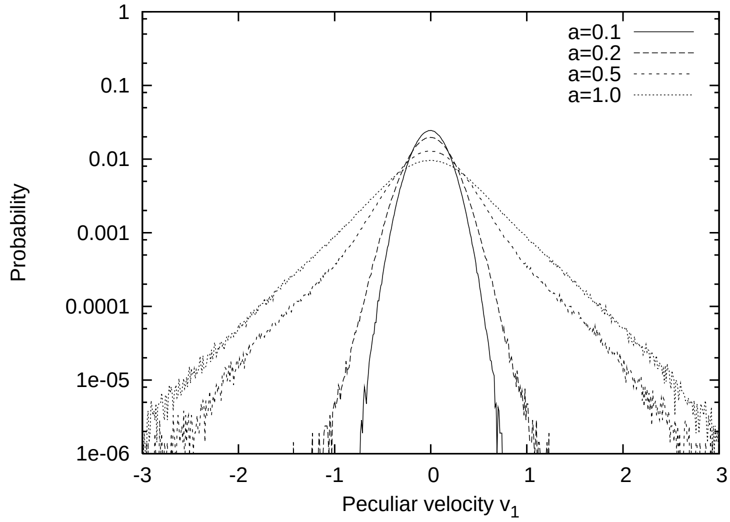

Figure 3. The PDF of the peculiar velocity in N-body simulation. During evolution, because of attraction to high density region, the PDF deviates from Gaussian distribution.

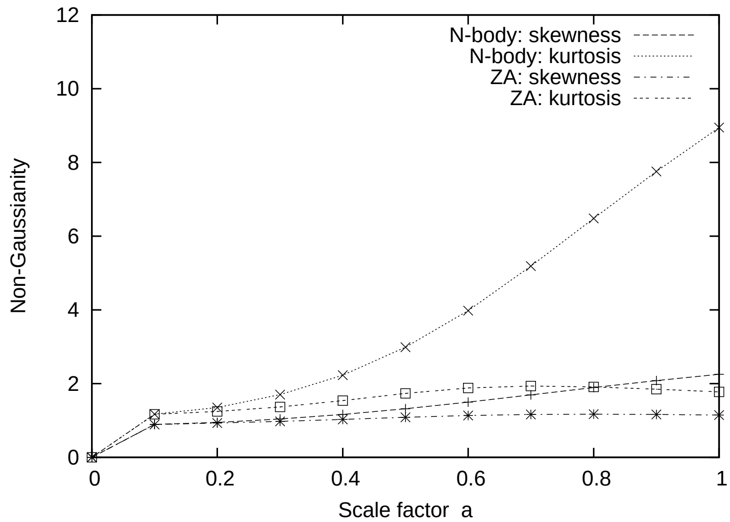

Figure 4. The skewness and the kurtosis of the density fluctuation. After quasi-nonlinear stage ($\delta_{\max} > 1$), the positive fluctuation evolves quickly. Then the skewness increases remarkably. Of course N-body simulation realizes strongly nonlinear evolution; the skewness and the kurtosis increase rapidly. For ZA, because of shell-crossing, the evolution of the skewness and the kurtosis stops gently.

we should choose an appropriate value for K_J. In this paper, following the results of our previous paper [34], we choose that the initial Jeans wavelength $K_J = 64$ at $a = 10^{-3}$. Here we do not specify a dark matter model; we just analyze the behavior of the pressure models. In the pressure model, the evolution of the perturbation depends on the scale. Therefore even if the initial distribution is Gaussian, we expect that the non-Gaussianity would appear during evolution.

Figure 6 shows the dispersion of the density fluctuation. In past papers [25, 26], we

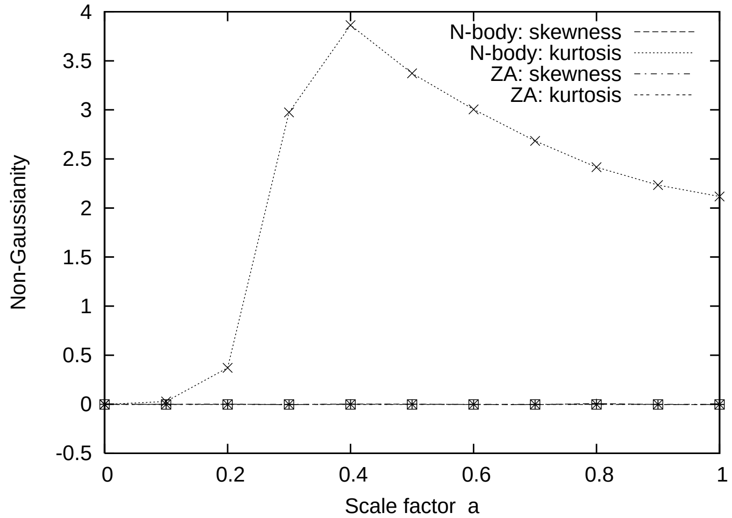

Figure 5. The skewness and the kurtosis of the peculiar velocity. Here we choose one direction of the peculiar velocity. In N-body simulation, the high-density region attracts surrounding matter, and the kurtosis increases. On the other hand, when we set an isotropic initial condition, the skewness almost does not increase. In ZA, because the evolution of the peculiar velocity is always linear, the PDF of the peculiar velocity remains Gaussian at all times.

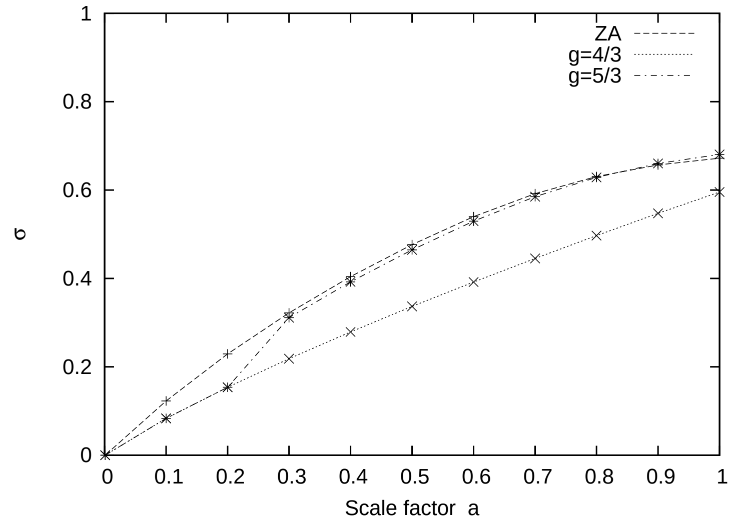

Figure 6. The dispersion of the density fluctuation in ZA and the pressure models. During evolution, the dispersion in the case of $\gamma = 5/3$ approaches to that in ZA.

show that the asymptotic behavior of the perturbative solution in the case of $\gamma = 5/3$ is similar to that in ZA. Here we show that the statistical quantities in the case of $\gamma = 5/3$ approach to these in ZA during evolution. As was done with the dust model, we analyze the quasi-nonlinear stage in the pressure model.

Figures 7 and 8 shows the skewness and the kurtosis of the peculiar velocity in the pressure model, respectively. Under the linear perturbation, for the peculiar velocity,

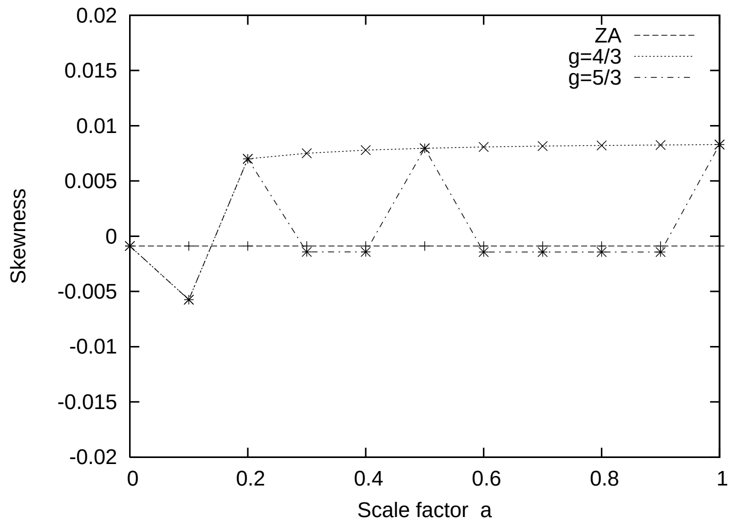

Figure 7. The skewness of the peculiar velocity in the pressure model. Here we choose one direction of the peculiar velocity. In the dust model, because the evolution of the peculiar velocity is always linear, the skewness takes a constant value. In the pressure model, although the non-Gaussianity appears during evolution, its extent remains small in the quasi-nonlinear stage.

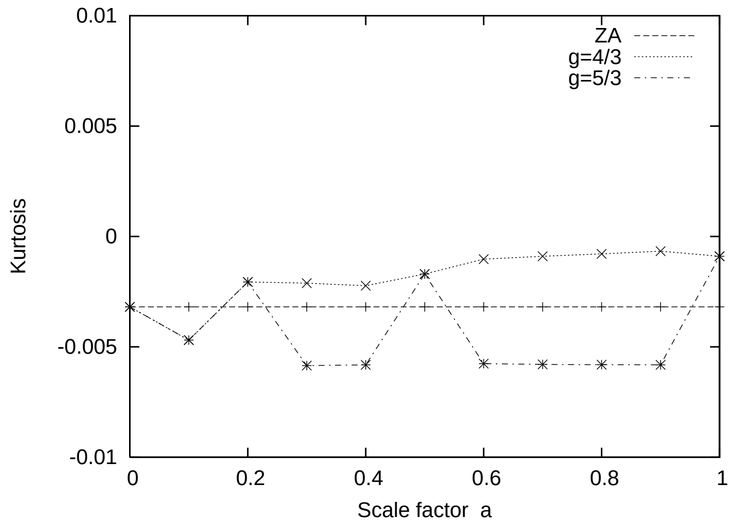

Figure 8. The kurtosis of the peculiar velocity in the pressure model. Here we choose one direction of the peculiar velocity. As in the case of the skewness, in the dust model, because the evolution of the peculiar velocity is always linear, the kurtosis takes a constant value. In the pressure model, although the non-Gaussianity appears during evolution, its extent remains small in the quasi-nonlinear stage.

the Lagrangian PDF is equal to the Eulerain PDF at all times [28]. In the pressure model, although the non-Gaussianity appears during evolution, its extent remains small in the quasi-nonlinear stage. In the dust model, the PDF of the peculiar velocity never changes. Even if the nonlinear structure forms, the motion of the matter does not stop. On the other hand, because the pressure especially affects the dense region, the motion

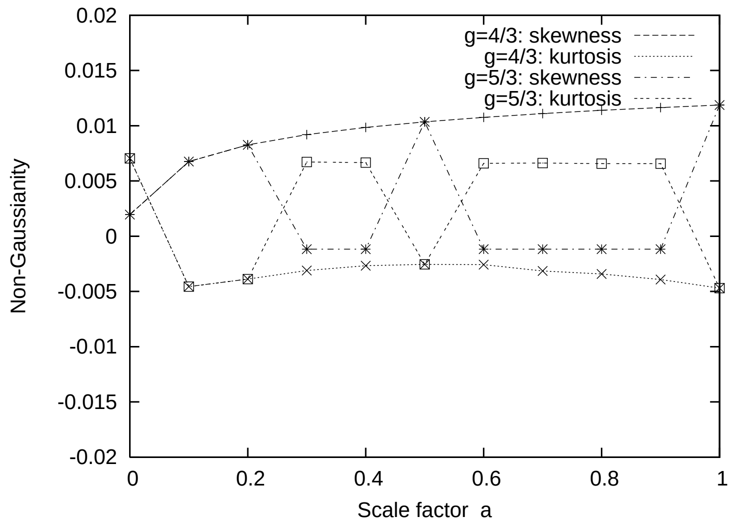

Figure 9. The skewness and the kurtosis of the density fluctuation in the Eulerian models. Although we consider the effect of the pressure, the PDF of the density fluctuation is almost Gaussian during evolution.

of the matter is slowed down. Therefore, as we show in Figure 7, the skewness of the peculiar velocity in the pressure model grows more than ten times bigger than that in ZA.

Next we show the skewness and the kurtosis of the density fluctuation in the pressure model. First we analyze the Eulerian linear approximation. Even if we consider the Eulerian linear approximation, the evolution of the fluctuation is not linear, because of the pressure. The evolution depends on the scale of the fluctuation. Therefore the PDF of the density fluctuation may deviate from Gaussian. Figure 9 shows the result. Although the skewness and the kurtosis oscillate, the value is still small in a nonlinear regime. Therefore we conclude that the PDF of the density fluctuation almost retains Gaussianity.

We analyze the PDFs in the Lagrangian linear approximation. As we show in equation (10), the relation between the density fluctuation and the Lagrangian displacement is nonlinear. Therefore as we show in the case of the dust model, even if we consider the linear approximation, the PDF of the density fluctuation deviates from Gaussian.

Figures 10 and 11 show the skewness and the kurtosis of the density fluctuation. In the dust model, because of shell-crossing, the growth of the skewness stops gently. In the pressure model, although the growth of the skewness lasts longer than in the case of the dust model, it finally stops because of shell-crossing. The behavior of the growth of the non-Gaussianity depends on the "equation of state."

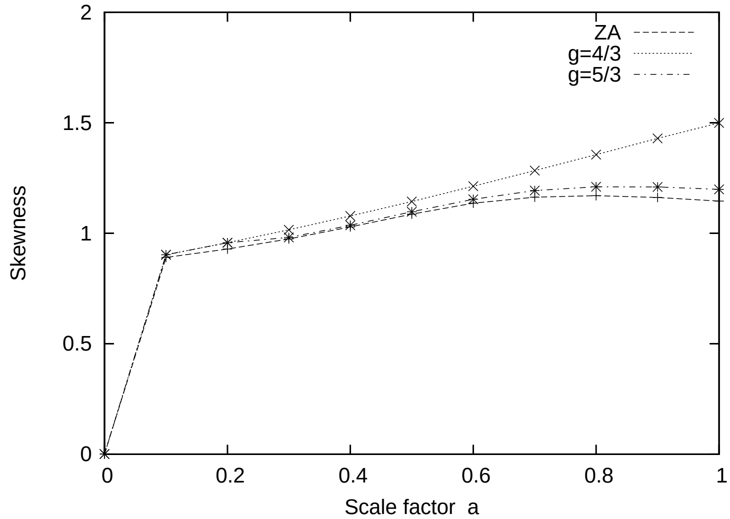

Figure 10. The skewness of the density fluctuation in the Lagrangian models. In the dust model, because of shell-crossing, the growth of the skewness stops gently. In the pressure model, although the growth of the skewness lasts longer than in the case of the dust model, it finally stops because of shell-crossing.

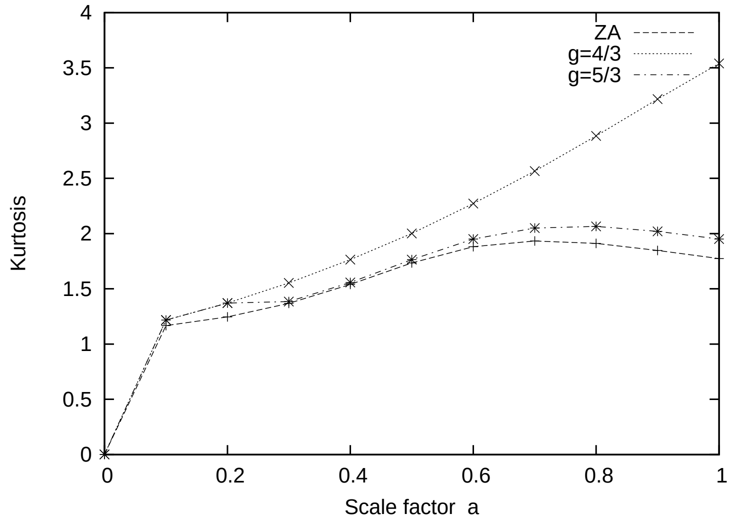

Figure 11. The kurtosis of the density fluctuation in the Lagrangian models. As in the case of the skewness, in the dust model, because of shell-crossing, the growth of the kurtosis stops gently. In the pressure model, although the growth of the kurtosis lasts longer than in the case of the dust model, it finally stops because of shell-crossing.

4. Summary

We analyzed a one-point PDF for the peculiar velocity and the density fluctuation. Here we start from the Gaussian distribution and analyze the occurrence of non-Gaussianity. For the dust model, in Eulerian linear approximation, because the growth of the fluctuation is always linear, the distribution always remains Gaussian. In Lagrangian linear approximation, although the PDF of the density fluctuation becomes non-Gaussian, the PDF of the peculiar velocity remains Gaussian, because the peculiar velocity is proportional to the Lagrangian displacement. Therefore the non-Gaussianity is produced by nonlinear effect.

Next we consider the effect of the pressure. If the pressure exerts an effect, the evolution of the fluctuation depends on the effect's scale. Therefore even if we take only the Eulerian linear approximation, the pressure may produce non-Gaussianity of the density fluctuation. However, from our results, the PDF of the density fluctuation in the Eulerian linear approximation is almost Gaussian during evolution. In the same way, the PDF of the peculiar velocity is almost Gaussian during evolution. Therefore we can conclude that most of the non-Gaussianity of the PDF is produced by the nonlinear effect of the perturbation.

We compared the evolution of the non-Gaussianity in the Lagrangian linear approximation. As we showed in Figures 10 and 11, the evolution depends on the "equation of state." In analyses of the PDFs of both the peculiar velocity and the density fluctuation, we can distinguish the polytropic index. According to past analyses [25, 26], the behavior of the linear perturbative solutions differ widely between the cases of $\gamma = 4/3$ and $\gamma = 5/3$. In the case of $\gamma = 4/3$, the behavior of the solutions strongly depends on the scale of the fluctuation. If we consider large-scale fluctuation, because self-gravity dominates, the fluctuation grows. On the other hand, if we consider small-scale fluctuation, because the pressure dominates, the fluctuation oscillates and decays. In the case of $\gamma = 5/3$, the pressure affects only the early stage. At the late time, the behavior of the perturbative solution approaches that of ZA solutions. Therefore the statistical quantities we computed, i.e., the variance, the skewness and the kurtosis of the peculiar velocity and the density fluctuation approaches to those in the case of ZA (Figures 6, 7, 8, 10 and 11).

Although our result can apply until quasi-nonlinear stage is reached, we can expect that an "equation of state" can be distinguished from the growth of the non-Gaussianity. If we can observe the PDF of the density fluctuation in high-z region, i.e., the quasi-nonlinear region, we can find the constraint of the character of the dark matter [35].

For comparing between the theoretical models and the observation, spacial two-point correlation function [36, 37, 38, 39, 40, 41, 42] can also be important. We will compute the correlation function and discuss the difference between the theoretical models.

Acknowledgments

We are grateful to Kei-ichi Maeda for his continuous encouragement. We thank Shuntaro Mizuno for useful discussion. This work was supported by the Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Culture, Sports, Science and Technology, Japan (Young Scientists (B) 16740152).

References

[1] Padmanabhan T 1993 *Structure Formation in the Universe* (Cambridge: Cambridge University Press)

[2] Coles P and Lucchin F 1995 *Cosmology: The Origin and Evolution of Cosmic Structure* (Chichester: John Wiley and Sons)

[3] Peacock J 1999 *Cosmological Physics* (Cambridge: Cambridge University Press)

[4] Zel'dovich Ya B 1970 *Astron. Astrophys.* **5** 84

[5] Arnol'd V I, Shandarin S F and Zel'dovich Ya B 1982 *Geophys. Astrophys. Fluid Dynamics* **20** 111

[6] Shandarin S F and Zel'dovich Ya B 1989 *Rev. Mod. Phys.* **61** 185

[7] Buchert T 1989 *Astron. Astrophys.* **223** 9

[8] Sahni V and Coles P 1995 *Phys. Rep.* **262** 1

[9] Jones B J T, Martínez V J, Saar E and Trimble V 2004 *Rev. Mod. Phys.* **76**, 1211

[10] Tatekawa T 2004 *Preprint* astro-ph/0412025

[11] Padmanabhan T 2005 *Preprint* gr-gc/0503107

[12] Munshi D, Sahni V and Starobinsky A A 1994 *Astrophys. J.* **436** 517

[13] Sahni V and Shandarin S F 1996 *Mon. Not. R. Astron. Soc.* **282** 641

[14] Yoshisato A, Matsubara T and Morikawa M 1998 *Astrophys. J.* **498** 48

[15] Gurbatov S N, Saichev A I and Shandarin S F 1989 *Mon. Not. R. Astron. Soc.* **236** 385

[16] Coles P, Melott A L and Shandarin S F 1993 *Mon. Not. R. Astron. Soc.* **260** 765

[17] Melott A L, Pellman T F and Shandarin S F 1994 *Mon. Not. R. Astron. Soc.* **269** 626

[18] Binney J and Tremaine S 1987 *Galactic Dynamics* (Princeton: Princeton University Press)

[19] Buchert T and Domínguez A 1998 *Astron. Astrophys.* **335** 395

[20] Buchert T, Domínguez A and Pérez-Mercader J 1999 *Astron. Astrophys.* **349** 343

[21] Domínguez A 2000 *Phys. Rev.* **D62** 103501

[22] Domínguez A 2002 *Mon. Not. R. Astron. Soc.* **334** 435

[23] Buchert T and Domínguez A 2005 *Preprint* astro-ph/0502318

[24] Adler S and Buchert T 1999 *Astron. Astrophys.* **343** 317

[25] Morita M and Tatekawa T 2001 *Mon. Not. R. Astron. Soc.* **328** 815

[26] Tatekawa T, Suda M, Maeda K, Morita M and Anzai H 2002 *Phys. Rev.* **D66** 064014

[27] Tatekawa T 2005 *Phys. Rev.* **D71** 044024

[28] Kofman L, Bertschinger E, Gelb J M, Nusser A and Dekel A 1994 *Astrophys. J.* **420** 44

[29] Weinberg S 1972 *Gravitation and Cosmology* (New York: John Wiley and Sons)

[30] Peebles P J E 1980 *The Large-Scale Structure of the Universe* (Princeton: Princeton University Press) p 151

[31] Padmanabhan T and Subramanian K 1993 *Astrophys. J.* **410** 482

[32] Kayo I, Taruya A, and Suto Y 2001 *Astrophys. J.* **561** 22

[33] Ma C P and Bertschinger E 1995 *Astrophys. J.* **455** 7

[34] Tatekawa T 2004 *Phys. Rev.* **D69** 084020

[35] Ostriker J P and Steinhardt P 2003 *Science* **300** 1909

[36] Totsuji H and Kihara T 1969 *Pub. Astron. Soc. Japan* **21** 211

[37] Peebles P J E 1974 *Astron. Astrophys.* **32** 197

[38] Geller M J and Huchra J P 1989 *Science* **246** 897

[39] Jing Y P, Mo H J and Börner G 1998 *Astrophys J.* **494** 1

[40] Guzzo L *et al.* 2000 *Astron. Astrophys.* **355** 1

[41] Hawkins E *et al.* 2003 *Mon. Not. R. Astron. Soc.* **346** 78

[42] Zehavi I *et al.* 2002 *Astrophys. J.* **571** 192